

# Ohm's law revision

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The standard ohmic measurements by means of two extra leads contain an additional thermal correction to resistance. The current results in heating(cooling) at first(second) sample contact due to Peltier effect. The contacts temperatures are different. The measured voltage is the sum of the ohmic voltage swing and Peltier effect induced thermopower which is *linear* on current. As a result, the thermal correction to resistance measured exists at  $I \rightarrow 0$ . The correction should be in comparison with ohmic resistance. Above some critical frequency dependent on thermal inertial effects the thermal correction disappears.

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As well known the ohmic measurements (Fig.1) are carrying out at low current density in order to prevent heating. Usually, the only Joule heat is considered to be important. We recall, in contrast to Joule heat, the Peltier and Thomson effects are linear on current. The crucial point of the present paper is that the linear on current Peltier effect influences the ohmic measurements and results in the correction to resistance measured. It's well known that the one of the sample contacts is heated while the another one is cooled due to current induced Peltier effect. The temperature gradient established is proportional to current. In this case the Thomson heat is proportional to second power on current, hence, could be neglected. Finally, the voltage swing across the circuit includes Peltier effect induced thermopower which is linear on current. Accordingly, there exists the thermal correction to ohmic resistance of the sample.

Let us first consider an isotropic(or of cubic symmetry) conductor which can be in the thermodynamic non-equilibrium with respect to conducting electrons. In general, for inhomogeneous conductor the current density,  $\mathbf{j}$ , and energy flux density,  $\mathbf{q}$ , are given by the following equations <sup>[1]</sup>.

$$\mathbf{j} = \sigma(\mathbf{E} - \alpha \nabla T), \quad (1)$$

$$\mathbf{q} = (\phi + \alpha T)\mathbf{j} - \kappa \nabla T. \quad (2)$$

Here,  $\sigma$  is the conductivity,  $\alpha$  is the thermopower,  $\kappa$  is the thermal conductivity. For an inhomogeneous conductor the electrochemical potential,  $\phi = \varphi + \mu/e$ , is the sum of the electric potential,  $\varphi$ , and chemical potential,  $\mu$ , of the conducting electrons. Note, for homogeneous conductor the above definition of the potential results in an additional unimportant constant to  $\varphi$ , therefore, the average microscopic electric field,  $-\nabla\varphi$ , coincides with

$\mathbf{E} = -\nabla\phi$ . The first term in Eq.(1) corresponds to conventional Ohm's law, the second one describes the thermoelectric phenomena. For steady state

$$\text{div} \mathbf{j} = 0, \quad (3)$$

$$Q = -\text{div} \mathbf{q} = \text{div}(\kappa \nabla T) + j^2/\sigma - jT \nabla \alpha = 0, \quad (4)$$

where  $Q$  is the total amount of heat, evolved per unit time and volume of the conductor. The current flow is accompanied by both the Joule and Thomson heats which are proportional to the second(first) power of current respectively. Using Eqs.(1-4) one should find the potential,  $\phi(\mathbf{r})$ , and temperature,  $T(\mathbf{r})$ , for the conductor under given boundary conditions.

We now consider the thermal effects in connection with ohmic measurements (see Fig.(1)) of a conductor resistance. The current carrying conductor is connected by means of two identical extra leads to the current source (not shown). We assume that both contacts are ohmic ones. Then,  $\alpha$ ,  $\sigma$ ,  $\kappa$ , the length,  $l$ , and the conductor cross-section,  $S$ , are different for the leads and the sample. The voltage is measured between the open ends ("c" and "d") being kept at temperature,  $T_0$ , of an external thermal reservoir. In general, the contacts ("a" and "b") could be at different temperatures  $T_a$  and  $T_b$  respectively.

As well known the Peltier heat is generated by the current crossing the contact of two different conductors. At the contact( let say "a" in Fig.(1)) the temperature  $T$ , the electrochemical potential  $\phi$ , the normal components of the current  $I = jS$  and the total energy flux  $qS$ , are all continuous. Then, there exists the difference,  $\Delta\alpha = \alpha_1 - \alpha_2$ , of thermopowers. For  $\Delta\alpha > 0$  the charge intersecting the contact "a" gains the energy  $e\Delta\alpha T_a$ . Consequently,  $Q_a = I\Delta\alpha T_a$  is the amount of Peltier heat evolved per unit time in the contact "a". We underline that  $Q_a$  could be calculated by another way accounting the Thomson term in Eq.(4). Indeed,  $Q_a \equiv \int -IT \nabla \alpha dx$ , where the integration is accomplished over the contact length. In fact, Peltier effect is equivalent of Thomson effect established at the contact.

For  $\Delta\alpha > 0$  and current direction shown in Fig.(1) the contact "a" is heated while the contact "b" is cooled. Thus, the contacts are at different temperatures and  $T_a - T_b = \Delta T > 0$ . We now show that the standard ohmic measurements always result in the thermal correction to resistance measured. Using Eq.(1), the voltage swing,  $U$ , between the ends "c" and "d" is

$$U = \int_c^d (\mathbf{j}/\sigma + \alpha \nabla T) dx = RI + \varepsilon_T, \quad (5)$$

where  $R = 2R_1 + R_2 = 2l_1/(S_1\sigma_1) + l_2/(S_2\sigma_2)$  is the total resistance of the circuit. The first term in Eq.(5) corresponds to Ohm's law [2]. The second term,  $\varepsilon_T = \int_c^d \alpha dT$ , coincides with equation for conventional thermoelectromotive force under zero current conditions [1]. Note,  $\varepsilon_T$  is an universal value since for arbitrary cooling conditions it depends on the contact temperatures only. It's worth noting that there exists the correlation between thermoelectromotive force, Peltier and Thomson heats. Indeed, the total power evolved in the circuit,  $UI$ , is the sum of Joule heat  $RI^2$  and the thermal effects related power  $\varepsilon_T I$ . Then, the product  $\varepsilon_T I$  is exactly the sum of Peltier heat,  $Q_P = Q_a - Q_b = I\Delta\alpha\Delta T$ , evolved at both contacts and Thomson heat,  $Q_T = -\int_c^d IT\nabla\alpha dx$  in the conductors bulk:

$$I\varepsilon_T = Q_P + Q_T. \quad (6)$$

According to Eq.(6), for arbitrary circuit under the same contacts temperatures ( $T_a$ ,  $T_b$  and  $T_0$ ) the zero current measurements of thermoelectromotive force allow to find the total amount of both the Peltier and Thomson heats at  $I \neq 0$ .

We recall that the sample contacts are always extra heated(cooled) because of Peltier effect. The difference of contact temperatures  $\Delta T$  is linear on current, thus, there exists the thermal correction to ohmic resistance  $\Delta R = \varepsilon_T/I = U/I - R$ . For simplicity, we further assume that the conductivity,  $\sigma$ , thermopower,  $\alpha$ , and the thermal conductivity,  $\varkappa$ , are all temperature independent. In that case, the thermopower is given  $\varepsilon_T = \Delta\alpha\Delta T$ .

Using Eqs.(4,5) one could easily find out voltage swing  $U$  and, thus, thermal correction  $\Delta R$  for arbitrary circuit. Note, the real cooling conditions strongly influence  $\Delta R$ . We now precise the cooling conditions of the circuit shown in Fig.(1). Let us consider the adiabatic conditions when the sample is thermally isolated with respect to environment. As an example, the sample could be placed into the vacuum chamber(see Fig.1) surrounded by thermal reservoir kept at  $T_0$ . Then, we further neglect the heat transfer within the leads. Actually, that means the sample is self-isolated. We emphasize that under the above conditions the sample is not heated. Really, under zero on current approximation  $T_a, T_b \approx T_0$ , hence, the amount Peltier heat evolved at the first contact "a" is equal to one absorbed at the second contact "b". At both contacts the energy flux  $qS$  is continuous, thus

$$Q_a = -Q_b = I\Delta\alpha T_0 = -\varkappa_2 \frac{dT}{dx} S_2. \quad (7)$$

Using Eq.(7) one could find out immediately the thermal correction to resistivity as follows

$$\Delta R = \frac{T_0(\Delta\alpha)^2 l_2}{S_2 \varkappa_2}. \quad (9)$$

According to Eq.(5,9)  $\Delta R$  depends on reservoir temperature, geometry and heat conductivity of sample. We underline that the thermal correction is always positive, since the total amount of Peltier heat  $Q_P = \Delta R I^2 > 0$ .

Let us estimate the magnitude of the thermal correction to resistivity  $\Delta R$ , when both the conductor and leads are metals. At room temperature the electron heat conductivity and thermopower of electron gas are:  $\varkappa = L\sigma T$ ,  $\alpha = \frac{\pi^2 k}{2e} \xi$ , where  $L = \frac{\pi^2 k^2}{3e^2}$  is the Lorentz number,  $\xi = kT/E_f \ll 1$  is degeneracy parameter. The difference  $\Delta\alpha$  is of the order  $\frac{k\xi}{e}$ . Under the above assumptions one should easily find  $\Delta R/R \sim \xi^2 \ll 1$ . Thus, thermal correction is small compared with ohmic resistance because of degeneracy of the electron gas. Note for semimetals (bismuth,  $E_f \sim 35\text{meV}$ ) the thermal correction could be greater. In contrast to metal, the thermal correction should be  $\xi^{-2}$  times higher for non-degenerated semiconductor since in that case  $\Delta\alpha \approx \alpha_2 = \frac{k}{e}(\frac{5}{2} + r - \xi^{-1}) \sim k/e$ . Here,  $r = 3/2$  is the parameter related to phonon scattering mechanism. Let us consider non-degenerated n-InSb at  $T=0.5\text{K}$ . The Fermi energy lies between the conduction band and shallow donor impurity level  $\Delta E_d \approx 7\text{K}$ . Accordingly,  $\Delta\alpha = 11\frac{k}{e}$ . Then, at low temperatures the electron heat conductivity is less than phonon related Debye one  $\varkappa_{ph} = 0.05 \cdot T^3 \text{W/cmK}$ . Finally, for n-InSb with electron concentration  $n = 10^{13} \text{cm}^{-3}$  and mobility  $\mu = 5 \cdot 10^6 \text{cm}^2/\text{Vs}$  one obtains the thermal correction to resistivity  $\Delta R/R \sim 0.01$ .

In reality, the cooling conditions could be different from ones assumed. We now consider more realistic case when the local cooling of the sample is important. For example, the sample chamber should contain the gas. One could take into account the cooling effects using Eq.(4) with the linear term,  $-\beta(T - T_0)$ , included. Here,  $\beta$  denotes the strength of the sample-to-gas thermal exchange. Finally, under zero on current approximation the temperature downstream the sample(see Fig.(2, insert)) is given

$$T(\eta) = \frac{(T_a - T_0) \sinh[\lambda(1 - \eta)] + (T_b - T_0) \sinh[\lambda\eta]}{\sinh[\lambda]} + T_0, \quad (10)$$

where  $\eta = x/l_2$  is the dimensionless coordinate. According to Eq.(10), the sample local cooling is governed by the dimensionless parameter  $\lambda = \sqrt{\frac{\beta}{\varkappa_2}} l_2$ . Actually,  $\lambda$  is the ratio of outgoing and internal(within the sample) heat fluxes. When  $\lambda \ll 1$  the local cooling could be neglected, hence,  $T(\eta)$  is linear(Fig.(2)). Then, in the opposite case  $\lambda \gg 1$  of intensive cooling  $T(\eta)$  dependence is sharp near the contacts.

The above result allow us to calculate the thermal correction to sample resistance. Using Eqs.(5,7,10) and omitting the cumbersome algebraic calculations one could calculate the thermal correction to resistance as follows

$$\Delta R = \frac{T_0(\Delta\alpha)^2 l_2}{S_2 \kappa_2} \cdot \frac{\tanh(\lambda/2)}{(\lambda/2)}. \quad (11)$$

For small cooling  $\lambda \rightarrow 0$  Eqs.(9,11) coincide. In the opposite case  $\lambda \rightarrow \infty$  of strong cooling the difference  $\Delta T$  and, thus, thermal correction decrease (see Fig.2).

We now estimate  $\Delta R$  given by Eq.(11) for natural air convection cooling. For sample with typical dimension  $d \sim \sqrt{S}$  the outgoing thermal flux is given  $\kappa_{gas} \frac{Nu}{S} (T - T_0)$ , where  $Nu \sim 10$  is the Nusselt number. For n-InSb sample (0.5×0.5×0.5cm) the heat conductivity is  $\kappa_{ph} = 0.15 \text{ W/Kcm}$  ( $T=293\text{K}$ ). Then, assuming the air heat conductivity  $\kappa_{gas} = 2.6 \text{ W/cmK}$  one should find  $\lambda = 14$ . Thus, the thermal correction to resistivity is approximately 14 times less compared to one in absence of convection.

We emphasize that both dc and ac ohmic measurements leads to thermal correction. However, at high frequency  $\Delta R$  diminishes due to thermal inertial effects. In fact, Eq.(9) are valid below some critical frequency  $f_{cr} = \chi/d^2$ , where  $\chi$  is the temperature diffusive coefficient of the sample. For example, At  $T=293\text{K}$  for metal conductor  $\chi = \kappa/C \simeq 10^2 \text{ cm}^2/\text{s}$ , where  $C$  is the calorific capacity of the electron gas. Then for typical metal conductor  $d \sim \sqrt{S} = 1\text{mm}$ , one obtain the critical frequency as  $f_{cr} = 10^4 \text{ Hz}$ . We suggest that the spectral dependence of thermal correction could be used to estimate the magnitude of the thermal correction.

In conclusion, the ohmic measurements of a conductor resistance contain the thermal correction caused by Peltier effect. The thermal correction always exists, while its magnitude depends on actual cooling conditions of the circuit. Above some critical frequency dependent on thermal inertial effects the thermal correction disappears.

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[1] L.D.Landau, E.M.Lifshits, Electrodynamics, Pergamon, New York, 1966

[2] Ohm's law: "The amount of current flowing in a circuit made up of pure resistances is directly proportional to the electromotive forces(voltages) impressed on the circuit and inversely proportional to the total resistance of the circuit." (1827).

FIG. 1. The circuit for standart ohmic measurements. The dashed square represents the sample chamber.

FIG. 2. The dimensionless  $T(\eta)$  dependence given by Eq.(10) for fixed current,  $\lambda=0;2;5;10$  and contact temperature difference  $\Delta T$  found at  $\lambda=0$ .



